Assignment 5

1. Approximate the integral $\int_{13}^{15}\sin(x)\,\mathrm{d}x$ using each of the approximations we described using $h = 1$ and then again using $h = 0.5$. These include Riemann sums, the trapezoidal rule, Simpson's rule, our O(h^5) centered rule, and the composite 3-point and 4-point rules. Are the approximations using centered interpolating polynomials as accurate as those using only points to the left?

From Calculus, you know the correct answer is cos(13) – cos(15) = 1.667134694309018 to sixteen significant digits. Which is the most accurate formula?

2. Suppose we have a function that is piecewise constant, but discontinuous, so that $f(a) = 1$ and $f(b) = 0$, and somewhere between a and b, the value of the function drops from 1 to 0. We don't know exactly when between a and b the function f drops from 1 to 0, so what is the minimum and maximum possible values of the integral $\int_{a}^{b} f(x)\,\mathrm{d}x$? Use each of the formulas that estimate the integral of a function over one interval, including the trapezoidal rule, the O(h^5) centered rule, the O(h^4) three-point backward rule and the O(h^5) four-point backward rule. Recall some values may be outside the range $[a, b]$, so assume $f(x) = 1$ for $x < a$ and $f(x) = 0$ for $x > b$.

Which formula would you say is the best approximation?

3. Given the readings from a sensor that are being taken periodically,

6.2615, 6.8847, 7.4471, 8.0392, 8.6836

where the last being the most recent reading, do the following with least-squares best-fitting linear polynomials:

a. approximate the value of the underlying signal at the time of the last reading,
b. estimate the value of the underlying signal one time step into the future,
c. estimate the rate of change of the underlying signal assuming the readings are being taken once every 10 seconds, and
d. estimate the integral over the most recent time step of the underlying signal, again, assuming the readings are being taken once every 10 seconds.

4. Given the readings from a sensor that are being taken periodically,

3.786, 3.2866, 2.5966, 1.6497, 0.5556

where the last is the most recent reading, do the following with least-squares best-fitting quadratic polynomials:

a. approximate the value of the underlying signal at the time of the last reading,
b. estimate the value of the underlying signal one time step into the future,
c. estimate the rate of change of the underlying signal assuming the readings are being taken once every 10 seconds, and
d. estimate the integral over the most recent time step of the underlying signal, again, assuming the readings are being taken once every 10 seconds.

5. In Question 4, what would be your best estimate as to when the underlying signal will be zero?

Acknowledgement: Sahil Ahmed Butt suggested wording over which time interval certain integrals were asked about were insufficiently clear. Additional wording has been added.